Solution to Running a Sub Shop

1 A Quick Introduction to Combinatorics

- Combinatorics is just a fancy way to talk about counting.
- In this particular case we are counting the number of possible combinations.
- One thing to keep in mind is that the number of choices for two independent categories multiply. In other words, I can choose the meat and the bread without one influencing the other, thus they are independent. If there is 4 ways to choose the bread and 3 ways to choose the meat then there are 12 ways to choose altogether (one I choose the bread I had 3 ways to choose the meat... each time). In a mathematical sentence (at least a combinatorics type) this is generally represented with an “and.”
- An “or” generally means addition in combinatorics.

To make our calculations easier, we will need to following notation:

\[
\binom{n}{r} := \frac{n!}{r!(n-r)!} \tag{1.1}
\]

where \(n! := n(n-1)(n-2)\ldots(2)\cdot 1 = 1\cdot 2\cdot 3\ldots(n-1)n\). \(nCr\) is read \(n\) choose \(r\) an representing the number of ways to choose \(r\) things out of a collection of \(n\) things. For example, if we need to choose a team of 3 people out of a group of 5, there are

\[
\frac{5!}{3!(2)!} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = \frac{5 \cdot 4 \cdot 3!}{3! \cdot 2!} = \frac{5 \cdot 4}{2} = 10.
\]

The definition does not care about the order the people have been chosen and thus does not work if we are picking a short stop, first baseman, and a pitcher since shuffling the people into different positions (called a permutation) gives more combinations.

2 Our Sub Shop

We will handle each of the categories separately and then multiply all of our results (since we can choose from each category without influencing our choices in the others).

First we will handle the bread:

- There are three bread options.
- We will assume that everyone chooses some type of bread.
- We can also assume that we will only use one type of bread per sandwich.
- Thus we have to pick one out of 3 options.
There are
\[ 3C_1 = \frac{3!}{1!(2)!} = 3 \text{ ways}. \]  \hspace{1cm} (2.1)

Next we consider the meats:

- There are four meat options.
- Now people can choose no meat, one meat, two meats, three meats, or even four meats since the only thing that we restricted was that they could not choose the same meat twice.

There are
\[ 4C_0 + 4C_1 + 4C_2 + 4C_3 + 4C_4 \text{ ways}. \]  \hspace{1cm} (2.2)

For ease of notation, we use a standard notation
\[ \sum_{i=0}^{4} \binom{4}{i} \]  \hspace{1cm} (2.3)

where the \( \sum_{i=0}^{4} \) represents the sum as we change the amounts that we are choosing. Adding up gives us 16 ways to choose the meats.

Now we consider the sauces:

- There are four sauce options.
- While people could technically choose up to all four sauces, with the sauces offered it is reasonable to restrict it to only up to two sauces.
- For those that really like to mix flavors we will include the number of ways to use up to four sauces in parentheses.

There are
\[ \sum_{i=0}^{2} \binom{i}{i} \text{ ways}. \]  \hspace{1cm} (2.4)

Adding up gives us 11 ways to choose the sauces (16 again if we allow up to four sauces).

Finally we consider the veggies:

- There are six veggies options.
- Again we are not allowing doubling up but people can still choose up to 6 veggies on a sandwich.

There are
\[ \sum_{i=0}^{6} \binom{6}{i} = 64 \text{ ways}. \]  \hspace{1cm} (2.5)

3 Putting Our Answer Together

- Now we need to multiply all the numbers from our calculations together to get the final number of possibilities.
- This gives us \( 3 \times 16 \times 11 \times 64 = 33,792 \) to make our sandwiches (49,152 if we allow up to four sauces).
- That certainly seems like quite a large number of possible combinations from such a small starting number of ingredients, but that is great news for our sub shop.